

Self-Lensing By A Stellar Disk

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ABSTRACT

I derive a general expression for the optical depth τ for gravitational lensing of stars in a disk by Massive Compact Objects (Machos) in the same disk. For the more restricted case where the disk is self-gravitating and the stars and Machos have the same distribution function, I find $\tau = 2 \langle v^2 \rangle / c^2 \sec^2 i$ where $\langle v^2 \rangle$ is the mass-weighted vertical velocity dispersion, and i is the angle of inclination. This result does not depend on any assumptions about the velocity distribution. As an example, if stars within the bar of the Large Magellanic Cloud (LMC) account for the observed optical depth $\tau \sim 8 \times 10^{-8}$ as has recently been suggested, then $v \gtrsim 60 \text{ km s}^{-1}$. This is substantially larger than the measured dispersions of known LMC populations.

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1. Introduction

Alcock et al. (1993) and Aubourg et al. (1993) have recently reported the detection of several candidate microlensing events toward the Large Magellanic Cloud (LMC). These discoveries have stimulated a number of authors to consider the lensing of the LMC stars by Massive Compact Objects (Machos) in the bar of the LMC itself (Wu 1994; Sahu 1994). Here I present some very general results on self-lensing by a distant galactic disk and apply these results to the LMC. The results can also be applied to lensing by the disk of M31 (e.g. Gould 1994b), and with some modification to self-lensing by the bulge of the Milky Way.

2. Self-Lensing By A Disk

Consider a galactic disk at a distance d that is large compared to the disk thickness. The optical depth to lensing of a disk star at distance $d + z$ is

$$\tau(z) = \frac{4\pi G}{c^2} \int_{-\infty}^z dy \rho_m(y \cos i) \frac{(z-y)(d+y)}{d+z} \rightarrow \frac{4\pi G}{c^2} \int_{-\infty}^z dy \rho_m(y \cos i)(z-y), \quad (2.1)$$

where i is the angle of inclination and $\rho_m(z)$ is the mass density of Machos as a function of height above the plane. The mean observed optical depth over the population of all stars in the disk is

$$\tau = \frac{4\pi G}{c^2} N_s^{-1} \int_{-\infty}^{\infty} dz n_s(z) \int_{-\infty}^z dy \rho_m(y \cos i)(z-y), \quad (2.2)$$

where n_s is the number density of sources and where I define

$$N_s \equiv \int_{-\infty}^{\infty} dz n_s(z), \quad \Sigma_m \equiv \int_{-\infty}^{\infty} dz \rho_m(z). \quad (2.3)$$

Equation (2.2) can be evaluated,

$$\tau = \frac{\pi G \Sigma_m}{c^2} \sec^2 i \int_{-\infty}^{\infty} dz [1 - F_m(z)][1 + F_s(z)], \quad (2.4)$$

where

$$F_m(z) \equiv -1 + \frac{2}{\Sigma_m} \int_{-\infty}^z dy \rho_m(y), \quad F_s(z) \equiv -1 + \frac{2}{N_s} \int_{-\infty}^z dy n_s(y). \quad (2.5)$$

For the most part, I will restrict attention to the case where ρ and n are symmetric in z , in which case

$$F_m(z) \equiv \frac{2}{\Sigma_m} \int_0^z dy \rho_m(y), \quad (2.6)$$

and similarly for $F_n(z)$, so that

$$\tau = \frac{2\pi G \Sigma_m}{c^2} \sec^2 i \int_0^{\infty} dz [1 - F_m(z)][1 + F_s(z)]. \quad (2.7)$$

For the case where the stars and Machos have the same distribution, equation (2.7) becomes

$$\tau = \frac{2\pi G \Sigma_m}{c^2} \sec^2 i \int_0^{\infty} dz \{1 - [F_m(z)]^2\} \quad (\text{pure self-lensing}). \quad (2.8)$$

On the other hand, suppose that the stars are confined to a thin planar distribution of much smaller scale height than the Machos. In this case $F_n(z) = \Theta(z)$ where Θ is the step function, so that

$$\tau = \frac{2\pi G \Sigma_m}{c^2} \sec^2 i \int_0^{\infty} dz [1 - F_m(z)] \quad (\text{thin stellar disk}). \quad (2.9)$$

Note that equation (2.9) is always smaller than equation (2.8).

3. Self-Gravitating Disk

Suppose that the source number distribution is proportional to the Macho mass distribution and let the disk potential, $\Psi(z)$ be due entirely to the Machos. Then the Jeans equation,

$$\frac{d[\rho_m(z)\overline{v^2(z)}]}{dz} = -\rho_m(z)\frac{d\Psi}{dz}, \quad (3.1)$$

can be used to evaluate the optical depth (2.8). In this case, $d\Psi/dz = 2\pi G\Sigma_m F_m(z)$, so that

$$\rho_m(z)\overline{v^2(z)} = \pi G\Sigma_m^2 \int^z dy F(y) \frac{dF}{dy} = \frac{\pi G\Sigma_m^2}{2} \{1 - [F(z)]^2\}, \quad (3.2)$$

where I have evaluated the integration constant using the fact that the pressure vanishes at infinity. Substituting equation (3.2) into equation (2.8), I find

$$\tau = 2 \frac{\langle v^2 \rangle}{c^2} \sec^2 i, \quad (3.3)$$

where $\langle v^2 \rangle$ is the mass weighted velocity dispersion,

$$\langle v^2 \rangle \equiv \frac{2}{\Sigma_m} \int_0^\infty dz \rho_m(z) \overline{v^2(z)}. \quad (3.4)$$

4. Some Additional Effects

Even if the stars and Machos have the same distribution, there are two effects which tend to reduce the optical depth relative to equation (3.3). First, the disk will in general have some mass (e.g., a thin sheet of gas) that provides neither sources nor lenses. Any such non-lensing mass distribution will increase $|F_m(z)|$ for all values of z and hence will, by equation (2.8) reduce the optical depth. The effect is not large however. In the most extreme case where there is a very large amount of such material near the plane, an isothermal population of stars would be distributed as $\rho_m(z) = (\Sigma_m/2h) \exp(-|z|/h)$, where h is the scale height. Then $F_m(z) = 1 - \exp(-|z|/h)$, so that $\tau = 3\pi G \Sigma_m h / c^2$. That is, $\tau = 1.5 \langle v^2 \rangle / c^2$, a 25% reduction relative to the pure self-gravitating disk.

Second, there is extinction. Here, the stars on the far side of the disk (which have higher optical depths than the near side) will be preferentially eliminated leading to lower optical depths. In M31, where $i \sim 78^\circ$ so that the dust at the midplane of the disk is probably opaque to optical light, the effect can be very large (Gould 1994b). From equation (2.4), I find that the optical depth is reduced by a factor of $\ln 2/2$ for an isothermal self-gravitating disk [$F_m(z) = \tanh(z/D)$], and by $1/3$ for an exponential disk. For the LMC, however, the effect is small.

5. Nearly Face-on Galaxies

As a practical matter, for nearly face-on galaxies one estimates the vertical velocity dispersion from the line of sight dispersion. If the velocity ellipsoids of other galaxies have axis ratios 4:3:2 like the Milky Way, then the line of sight dispersion overestimates the vertical dispersion, $\langle v_z^2 \rangle$ by a factor $\sim (1 + 2 \sin^2 i)$. Hence, for galaxies $i \lesssim 45^\circ$, the upper limit $\tau < 2 \langle v^2 \rangle / c^2 \sec^2 i$ derived in the previous two sections remains valid provided that one uses the line of sight dispersion to estimate the vertical dispersion.

6. Application to the LMC

The observed line of sight velocity dispersion of CH stars in the inner parts of the LMC is $\sim 20 \text{ km s}^{-1}$ (Cowely & Hartwick 1991). If the disk LMC stars have a similar dispersion, the upper limit to their contribution to the optical depth is $\tau < 1 \times 10^{-8}$. I have previously estimated (Gould 1994a) the observed optical depth to be $\tau \sim 8 \times 10^{-7}$ on the basis of the initial reports of Alcock et al. (1993). Subsequently, a larger area has been searched and two more events have been found (K. Griest 1994, private communication), but my estimate remains unchanged. Therefore, unless the general population of stars in the LMC have a substantially larger dispersion than the CH stars, they cannot contribute significantly to the observed optical depth. The suggestion of Sahu (1994) that the observed optical depth is due to stars in the LMC would require that the line-of-sight dispersion be $v_z \gtrsim 60 \text{ km s}^{-1}$, far in excess of the dispersion of any known population.

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